Chapter 2: Nonreciprocal Phase Shift for Magneto-Optical Isolator

2.1 Introduction
Optical nonreciprocal devices are indispensable to eliminate unwanted reflected light in the fiber link and protect optical active devices. The optical nonreciprocity is obtained from magneto-optic effect. A waveguide magneto-optical isolator employing nonreciprocal phase shift is investigated in this study. The nonreciprocal phase shift provides direction-dependent propagation constants due to the first-order magneto-optic effect with an external magnetic field applied transversally to the light propagation direction. A Mach-Zehnder interferometer configuration with the nonreciprocal phase shift realizes a magneto-optical isolator.

In this chapter, the nonreciprocal phase shift in a planar waveguide structure is analyzed by numerical calculation. Two numerical approaches are presented. One is solving directly the eigenvalue equation derived from the Maxwell equation with 1-dimensional (1-D) waveguide models. The other is a calculation using the perturbation theory with 2-dimensional (2-D) waveguide models. Then the principle of isolator operation and its design of the magneto-optical isolator are presented.

2.2 Theoretical analysis of nonreciprocal phase shift
The magneto-optic effects are described by the off-diagonal components of the dielectric permittivity tensor in a magneto-optic material [1]. The complex relative permittivity tensor of dielectric materials is given by

\[
\tilde{\varepsilon} = \begin{bmatrix}
\varepsilon_{xx} & \varepsilon_{yx} & \varepsilon_{zx} \\
\varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{zy} \\
\varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz}
\end{bmatrix},
\]  

(2.1)
As we consider loss-free material, the diagonal elements are real. The real and imaginary parts of the off-diagonal elements involve anisotropic and gyrotropic systems, respectively. Anisotropic systems are generated by the crystal-axis rotation in birefringent materials or the electro optic effect. Gyrotropic systems are generated by the magneto-optic effect. Since the corresponding elements such as $\varepsilon_{xy}$ and $\varepsilon_{yx}$ are complex conjugate to each other, the anisotropic systems are reciprocal due to the real part, and the gyrotropic systems are nonreciprocal due to the imaginary part. The imaginary off-diagonal elements are induced by an orthogonal magnetization. When the light propagates along the $z$-axis, the $\varepsilon_{xy}$ induced by $z$-aligned magnetization is related to the Faraday rotation and the $\varepsilon_{xz}$ or $\varepsilon_{yz}$ induced by $y$- or $x$-aligned magnetization are related to the nonreciprocal phase shift.

In this section, the nonreciprocal phase shift is theoretically analyzed. We assume a loss-free magneto-optic waveguide in which the magneto-optic film is located in the $x$-$z$ plane. The magnetization is aligned along the $x$-axis and lightwaves propagate along the $z$-axis. Here, a magneto-optic linear effect such as Cotton-Mouton effect is neglected. The relative permittivity of a magneto-optic film is then given by

$$
\tilde{\varepsilon} = \begin{bmatrix}
\varepsilon_x & 0 & 0 \\
0 & \varepsilon_y & j\gamma \\
0 & -j\gamma & \varepsilon_z
\end{bmatrix}
$$

(2.2)

where $\varepsilon_x$, $\varepsilon_y$, and $\varepsilon_z$ are related to the isotropic refractive indices denoted as $\varepsilon = n^2$ in the $x$, $y$, and $z$ direction, respectively. The off-diagonal elements $\gamma$ are proportional to the specific Faraday rotation coefficient $\Theta_F$ by

$$
\gamma = \frac{2n\Theta_F}{k_0}
$$

(2.3)

where $k_0$ is the wavenumber in vacuum.

The Maxwell equation is written as

$$
\nabla \times \mathbf{E} = -j\omega\mu_0\mathbf{H}
$$

(2.4)

$$
\nabla \times \mathbf{H} = j\omega\tilde{\varepsilon}\varepsilon_0\mathbf{E}
$$

(2.5)

$$
\nabla \cdot \mathbf{H} = 0
$$

(2.6)

$$
\nabla \cdot (\tilde{\varepsilon}\mathbf{E}) = 0
$$

(2.7)
where \( \omega \) is the angular frequency, \( \mathbf{E} \) and \( \mathbf{H} \) are the electric and magnetic field vectors, respectively, \( \varepsilon_0 \) and \( \mu_0 \) are the permittivity and permeability of free space, respectively. From (2.2), the Maxwell equation (2.4) and (2.5) are rewritten in component form as

\[
\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu_0 H_x, \tag{2.8}
\]

\[
\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu_0 H_y, \tag{2.9}
\]

\[
\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu_0 H_z, \tag{2.10}
\]

\[
\frac{\partial H_y}{\partial y} - \frac{\partial H_z}{\partial z} = j\omega\varepsilon_0 \varepsilon_x E_x, \tag{2.11}
\]

\[
\frac{\partial H_x}{\partial z} - \frac{\partial H_y}{\partial x} = j\omega\varepsilon_0 (\varepsilon_y E_y + j\gamma E_z), \tag{2.12}
\]

\[
\frac{\partial H_x}{\partial x} - \frac{\partial H_z}{\partial y} = j\omega\varepsilon_0 (\varepsilon_z E_z - j\gamma E_y). \tag{2.13}
\]

2.2.1 Solving eigenvalue equation with 1-D waveguide model

We consider a 1-dimensional waveguide model as shown in Fig. 2.1. The thickness of the guiding layer is defined \( d \). Since all layers are uniform along the \( x \)-axis, \( \partial / \partial x = 0 \).

![Fig. 2.1 Geometry of the three-layered slab waveguide model.](image-url)
First, TM mode propagating along the $z$-axis is examined. Then the propagating modes are described by

$$E = (0, E_y, E_z) \exp\{j(\omega t - \beta z)\}, \quad (2.14)$$

$$H = (H_x, 0, 0) \exp\{j(\omega t - \beta z)\} \quad (2.15)$$

where $\beta$ denotes the longitudinal propagation constants along the $z$-axis. From (2.12) and (2.13), $E_y$ and $E_z$ are expressed as

$$E_y = \frac{1}{\omega \varepsilon_0 (\varepsilon_x \varepsilon_z - \gamma^2)} \left( \gamma \frac{\partial H_z}{\partial y} - j \varepsilon_z \frac{\partial H_x}{\partial z} \right), \quad (2.16)$$

$$E_z = \frac{1}{\omega \varepsilon_0 (\varepsilon_x \varepsilon_z - \gamma^2)} \left( \gamma \frac{\partial H_x}{\partial z} + j \varepsilon_y \frac{\partial H_z}{\partial y} \right). \quad (2.17)$$

Substituting (2.16) and (2.17) into (2.8), we obtain the following wave equation for $H_x$

$$\frac{1}{\varepsilon_y \varepsilon_z - \gamma^2} \left( \varepsilon_y \frac{\partial^2 H_x}{\partial y^2} + \varepsilon_z \frac{\partial^2 H_x}{\partial z^2} \right) + k_0^2 H_x = 0 \quad (2.18)$$

where we use $k_0^2 = \omega^2 \varepsilon_0 \mu_0$. Using $\partial/\partial z = -j \beta$, (2.18) is rewritten as

$$\frac{\partial^2 H_x}{\partial y^2} + \left( k_0^2 \varepsilon' - \beta^2 \frac{\varepsilon_z}{\varepsilon_y} \right) H_x = 0 \quad (2.19)$$

with

$$\varepsilon' = \frac{\varepsilon_x \varepsilon_z - \gamma^2}{\varepsilon_y}. \quad (2.20)$$

For a guiding mode in the planar waveguide shown in Fig. 2.1, the effective refractive index $N (= \beta / k_0)$ must be

$$\varepsilon_{y1}, \varepsilon_{y3} < N^2 < \varepsilon_{y2} \quad (2.21)$$

and the wave vectors along $y$-axis are defined with

$$k_0^2 \varepsilon_i - \frac{\varepsilon_z}{\varepsilon_{yi}} \beta^2 = \begin{cases} -\eta_i^2 \quad (i = 1, 3) \\ k_2^2 \quad (i = 2) \end{cases} \quad (2.22)$$

One can solve the wave equation (2.19) for each layer by

$$H_y = \begin{cases} A_y \exp(-\eta_i (y - d)) \exp(-j \beta y) \quad (y \geq d) \\ (A_y \cos k_3 y + B_y \sin k_3 y) \exp(-j \beta y) \quad (0 \leq y < d) \\ A_y \exp(\eta_i y) \exp(-j \beta y) \quad (y < 0) \end{cases} \quad (2.23)$$
where \( A_1, A_2, B_2, \) and \( A_3 \) are the field amplitudes. Substituting (2.23) into (2.17), \( E_z \) for each layer is obtained as

\[
E_z = \begin{cases} 
\left( \frac{\gamma_1 \beta + \eta_i \varepsilon_{y_i} A_i}{j \omega \epsilon_0 \epsilon_{y_i}'} \right) \exp(-\eta_i (y - d)) \exp(-j\beta z) \\
\left( \frac{\gamma_2 \beta A_2 - \varepsilon_{y_2} k_2 B_2}{j \omega \epsilon_0 \epsilon_{y_2}'} \right) \exp(k_2 y) \cos k_2 y + \left( \frac{\gamma_2 ^2 \beta A_2 + \varepsilon_{y_2} k_3 A_3}{j \omega \epsilon_0 \epsilon_{y_3}'} \right) \sin k_3 y \\
\left( \frac{\gamma_3 \beta - \eta_3 \varepsilon_{y_3} A_3}{j \omega \epsilon_0 \epsilon_{y_3}'} \right) \exp(\eta_3 y) \exp(-j\beta z)
\end{cases}
\]  

Using the boundary condition that \( H_x \) and \( E_z \) are continuous across \( y=0 \) and \( y=d \), we obtain the following eigenvalue equation

\[
\tan k_2 d = \frac{k_2^2}{\varepsilon_2} \left\{ \left( \frac{\eta_i + \gamma_i \beta}{\varepsilon_i} \right) \frac{1}{\varepsilon_{y_i}'} + \left( \frac{\eta_3 - \gamma_3 \beta}{\varepsilon_3} \right) \frac{1}{\varepsilon_{y_3}'} \right\} - \frac{k_2^2}{\varepsilon_2} \left\{ \left( \frac{\eta_3 - \gamma_3 \beta}{\varepsilon_{y_3}'} \right) \frac{1}{\varepsilon_{y_3}'} + \left( \frac{\eta_1 + \gamma_1 \beta}{\varepsilon_1} \right) \frac{1}{\varepsilon_{y_1}'} \right\} \gamma_2 \beta \frac{1}{\varepsilon_{y_2}'} + \left( \frac{\gamma_2 \beta}{\varepsilon_{y_2}'} \right) \right\}
\]  

(2.25)

where \( \varepsilon_{y_i} \) is simplified as \( \varepsilon_i \) for each layer \( i=1,2,3 \). This equation can be solved by a numerical analysis. Because of the linear terms in \( \beta \) involving the off-diagonal elements \( \gamma \), there are nonreciprocal solutions for \( \beta \) as the propagation and magnetization directions. And the nonreciprocal phase shift is given by

\[
\Delta \beta = \beta_+ - \beta_-. 
\]  

(2.26)

Notice that, when the slab structure is symmetric denoted as \( \varepsilon_1=\varepsilon_3 \) and \( \gamma_1=\gamma_3 \), the linear terms in \( \beta \) vanish and \( \Delta \beta \) becomes zero. This means that the nonreciprocal phase shift is induced by the asymmetry of the waveguide structure transversally to the magnetization direction. The difference in the propagation constant means the difference in the distribution of electro-magnetic field depending on the propagation direction.

Next, TE mode propagating along the \( z \)-axis is examined. The propagating modes are described by

\[
E = \left( E_z, 0, 0 \right) \exp\left\{ j(\omega t - \beta z) \right\},
\]  

(2.27)

\[
H = \left( 0, H_y, H_z \right) \exp\left\{ j(\omega t - \beta z) \right\}.
\]  

(2.28)
Substituting (2.9) and (2.10) into (2.11), we obtain the following wave equation for $E_x$

$$\frac{\partial^2 E_x}{\partial y^2} + \left(k_0^2 \varepsilon_{xx} - \beta^2\right)E_x = 0.$$  \hspace{1cm} (2.29)

With the wave vectors along the $y$-axis

$$k_0^2 \varepsilon_{xx} - \beta^2 = \begin{cases} -\eta_i^2, & (i = 1,3) \\ k_i^2, & (i = 2) \end{cases}$$  \hspace{1cm} (2.30)

one can solve (2.29) for $E_x$ and $H_z$. Using the boundary condition for $E_x$ and $H_z$ across $y=0$ and $y=d$, we obtain the following eigenvalue equation

$$\tan k_d d = \frac{k_d (\eta_1 + \eta_3)}{k_d^2 - \eta_1 \eta_3}.$$  \hspace{1cm} (2.31)

Since there are no linear terms in $\beta$, nonreciprocal phase shift is not induced for TE mode propagations with the magnetization along the $x$-axis.

In the same way, we consider a four-layered waveguide model shown in Fig. 2.2. An interlayer with the thickness of $d_2$ is inserted above the guiding layer with the thickness of $d_3$. TM propagating modes are described as (2.14) and (2.15), and we obtain the same wave equation as (2.19). The wave vectors along the $y$-axis are defined with

$$k_0^2 \varepsilon_{yy} - \nu^2 = \begin{cases} -\eta_i^2, & (i = 1,4) \\ \pm \eta_i^2, & (i = 2) \\ k_3^2, & (i = 3) \end{cases}$$  \hspace{1cm} (2.32)

where the sign of $i=2$ is positive when the interlayer works as a part of core layer as $\varepsilon_i > N^2$ and negative when it works as a cladding layer as $\varepsilon_i < N^2$. The solutions for $H_x$ are given by

$$H_x = \begin{cases} A_1 \exp[-\eta_1(y - d_2 - d_3)]\exp(-j\beta z), & (y \geq d_2 + d_3) \\ [A_2 \exp[\eta_2(y-d_3)] + B_2 \exp[-\eta_2(y-d_3)]]\exp(-j\beta z), & (d_3 \leq y < d_2 + d_3) \\ (A_3 \cos k_3 y + B_3 \sin k_3 y)\exp(-j\beta z), & (0 \leq y < d_3) \\ A_4 \exp(\eta_4 y)\exp(-j\beta z), & (y < 0) \end{cases}$$  \hspace{1cm} (2.33)

Substituting (2.33) into (2.17), $E_z$ for each layer is obtained. Using the boundary condition that $H_x$ and $E_z$ are continuous across $y=0$, $y=d_2$ and $y=d_2+d_3$, we obtain the following eigenvalue equation.
\[
\left( \frac{\eta_4 - \frac{\gamma_4 \beta}{\epsilon_4}}{\epsilon_4'} \right) + \frac{\gamma_3 \beta}{\epsilon_3'} \frac{1}{\epsilon_3'} C_0 = 0
\]

(2.34)

where

\[
C_0 = \begin{bmatrix}
\left( \eta_2 - \frac{\gamma_2 \beta}{\epsilon_2} \right) \frac{1}{\epsilon_2'} - \left( \eta_2 + \frac{\gamma_2 \beta}{\epsilon_2} \right) C_1 \\
\left( \eta_2 - \frac{\gamma_2 \beta}{\epsilon_2} \right) \frac{1}{\epsilon_2'} + \left( \eta_2 + \frac{\gamma_2 \beta}{\epsilon_2} \right) C_1
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{1}{1 + C_1} \cos k_3 d_3 + \frac{\gamma_3 \beta}{\epsilon_3'} \cos k_3 d_3 + \frac{k_3}{\epsilon_3'} \sin k_3 d_3 \\
\frac{1}{1 + C_1} \sin k_3 d_3 - \frac{\gamma_3 \beta}{\epsilon_3'} \sin k_3 d_3 + \frac{k_3}{\epsilon_3'} \cos k_3 d_3
\end{bmatrix}
\]

(2.35)

and

\[
C_1 = \begin{bmatrix}
\left( \eta_1 + \frac{\gamma_1 \beta}{\epsilon_1} \right) \frac{1}{\epsilon_1'} - \left( \eta_1 - \frac{\gamma_1 \beta}{\epsilon_1} \right) C_1 \\
\left( \eta_1 + \frac{\gamma_1 \beta}{\epsilon_1} \right) \frac{1}{\epsilon_1'} + \left( \eta_1 - \frac{\gamma_1 \beta}{\epsilon_1} \right) C_1
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{1}{\epsilon_1} \exp(2\eta_2 d_2) \left( \eta_1 + \frac{\gamma_1 \beta}{\epsilon_1} \right) \frac{1}{\epsilon_1'} - \left( \eta_1 + \frac{\gamma_1 \beta}{\epsilon_1} \right) \frac{1}{\epsilon_1'}
\end{bmatrix}
\]

(2.36)

This equation can be solved by a numerical analysis. The linear terms in \( \beta \) involving the off-diagonal elements \( \gamma \) induce nonreciprocal solutions for \( \beta \), and the nonreciprocal phase shift \( \Delta \beta \) is obtained.

![Fig. 2.2 Geometry of the four-layered slab waveguide model.](image-url)
2.2.2 Perturbation theory with 2-D waveguide model

We consider a 2-dimensional waveguide model as shown in Fig. 2.3. In this case, the wave equation for $H_x$ derived from the Maxwell equation (2.4)-(2.7) has differential terms $\partial x$, $\partial y$ with respect to $x$ and $y$. The 2-D Maxwell equation considering $\partial / \partial x$ and $\partial / \partial y$ with the magneto-optic effect is a complex system even though a directly solution by finite element method is reported [2]. Easier way to obtain the nonreciprocal phase shift is using perturbation theory [1,3,4]. When the off-diagonal elements $\gamma$ of the permittivity tensor are much smaller than the diagonal elements $\varepsilon$, the gyrotropic effect can be treated as a perturbation in an unperturbed system without magnetization. The nonreciprocal propagation constants are given by

$$\beta_\pm = \beta_0 \pm \frac{\Delta \beta}{2}$$

(2.37)

where $\beta_0$ is the propagation constant of the unperturbed system. In perturbation theory, the nonreciprocal characteristic is given by

$$\Delta \beta = \frac{2\omega \varepsilon_0}{P} \iint E^* \Delta \varepsilon E dxdy$$

(2.38)

normalized by the power flow in the $z$ direction

$$P = \left( \iint \mathbf{E} \times \mathbf{H}^* + \mathbf{E}^* \times \mathbf{H} \right) dxdy .$$

(2.39)

$\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields of the unperturbed system and $\Delta \varepsilon$ is the perturbation induced by the magnetization.

Using the semi-vector approximation, TM modes propagating along the $z$-axis are described by

$$E = (0, E_y, E_z) \exp \{ j(\omega t - \beta z) \},$$

(2.40)

$$H = (H_x, 0, 0) \exp \{ j(\omega t - \beta z) \} .$$

(2.41)

From (2.12) and (2.13), $E_y$ and $E_z$ of the unperturbed system are expressed as

$$E_y = -\frac{j}{\omega \varepsilon_0 \varepsilon_y} \frac{\partial H_x}{\partial z},$$

(2.42)

$$E_z = \frac{j}{\omega \varepsilon_0 \varepsilon_z} \frac{\partial H_x}{\partial y} .$$

(2.43)
Then a magnetization along the $x$-axis yields the perturbation along the $y$-axis for TM modes, and the nonreciprocal phase shift is given by

$$\Delta \beta = \frac{2\omega \varepsilon_0}{P} \iint \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & j\gamma \\ 0 & -j\gamma & 0 \end{bmatrix} dxdy$$

$$= \frac{2\omega \varepsilon_0}{P} \iint \left\{ E_y^* (j\gamma) E_z + E_z^* (-j\gamma) E_y \right\} dxdy$$

$$= \frac{2\omega \varepsilon_0}{P} \iint 2 \Re \{ j\gamma E_y^* E_z \} dxdy$$

$$= \frac{2\gamma \beta}{\omega \varepsilon_0 \varepsilon_y \varepsilon_z} \iint H_x \frac{\partial H_x}{\partial y} dxdy,$$

(2.44)

$$P = \iint \{ E_y H_x^* + H_x^* E_y \} dxdy$$

$$= 2 \Re \iint E_y^* H_x dxdy = \frac{2\beta}{\omega \varepsilon_0 \varepsilon_y} \iint |H_x|^2 dxdy.$$  (2.45)

The integral term of $y$ in (2.44) exists only across the interfaces of the magneto-optic layer at $y=y_m$ as

$$\int \frac{\gamma}{\varepsilon_y \varepsilon_z} \frac{\partial H_x}{\partial y} dy = \left[ \frac{\gamma}{\varepsilon_y \varepsilon_z} H_x \right]_{y_m-0} - \left[ \frac{\gamma}{\varepsilon_y \varepsilon_z} H_x \right]_{y_m+0}.$$  (2.46)

Finally, we obtain

$$\Delta \beta = \sum_m \nu \int \gamma (\varepsilon_y \varepsilon_z)^{-1} |H_x(y_m)|^2 dx$$

$$\iint E_y^{-1} |H_x|^2 dxdy$$

(2.47)

where $\nu$ is $+1$ or $-1$ determined by the position of magneto-optic layer in (2.46).

The $|H_x|$ of the unperturbed system can be calculated by numerical simulation such as a beam propagation method (BPM) or a finite element method (FEM). The numerator of (2.47) is obtained from the calculation of the integral of off-diagonal elements, permittivity tensors, and simulated magnetic fields along the magneto-optic interface. The denominator of (2.47) is obtained from the integral of permittivity tensors and simulated fields all over the simulation region.
2.2.3 Calculation results and comparison

In this study, a cerium-substituted yttrium iron garnet CeY$_2$Fe$_5$O$_{12}$ (Ce:YIG) grown on a (Ca,Mg,Zr)-doped GGG substrate (SGGG) is used as the magneto-optic garnet. The Faraday rotation coefficient $\Theta_F$ of Ce:YIG is $-4500$ deg/cm, which corresponds to the off-diagonal permittivity $\gamma$ of 0.008525, at $\lambda=1.55$ $\mu$m [5]. As described in Chapter 1, our group has proposed the magneto-optical isolator employing nonreciprocal phase shift with some material systems. Here, we consider three kinds of waveguide structure as shown in Fig. 2.4, (a)SiO$_2$/Ce:YIG/SGGG, (b)Ce:YIG/GaInAsP/InP, and (c)Ce:YIG/Si/SiO$_2$ -layered structures, respectively. Rib waveguides are formed on the guiding layer to confine lightwave laterally. The latter two structures have side-cladding layer of air so that they are realized by a direct bonding [6-8]. The refractive indices of the material at $\lambda=1.55$ $\mu$m are shown in Table 2.1.

![Fig. 2.3 Geometry of the 2-D waveguide model.](image)

![Fig. 2.4 Waveguide structures for magneto-optical isolator.](image)
Table 2.1 Refractive indices at $\lambda = 1.55 \, \mu m$.

<table>
<thead>
<tr>
<th>Material</th>
<th>Refractive index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ce:YIG</td>
<td>2.20</td>
</tr>
<tr>
<td>SGGG</td>
<td>1.94</td>
</tr>
<tr>
<td>GaInAsP ($\lambda_g = 1.42 , \mu m$)</td>
<td>3.45</td>
</tr>
<tr>
<td>InP</td>
<td>3.17</td>
</tr>
<tr>
<td>Si</td>
<td>3.48</td>
</tr>
<tr>
<td>SiO$_2$</td>
<td>1.44</td>
</tr>
</tbody>
</table>

Figure 2.5 shows calculation results of the nonreciprocal phase shift as a function of waveguide height $h$. Dashed lines are calculated by solving eigenvalue equation with 1-D slab waveguide model where the side cladding layer is neglected. Solid lines are calculated by the perturbation theory with 2-D waveguide model where the unperturbed magnetic field is simulated by the mode solving via imaginary distance BPM [9,10]. In the semi-vector simulation, the magnetic fields are derived from the electric fields. For all the structure, the waveguide width $w$ and rib height $r$ are fixed at 2.0 $\mu m$ and 0.1 $\mu m$, respectively. The simulation grid sizes along $x$- and $y$-axis are $\Delta x = 0.05 \, \mu m$ and $\Delta y = 0.01 \, \mu m$, respectively, and the slice step along $z$-axis is $\Delta z = 0.005 – 0.05 \, \mu m$. There is little difference in the results between two methods, solving eigenvalue equation and perturbation theory. It is observed that a large nonreciprocal phase shift is obtained from the waveguide structure with Ce:YIG/Si/SiO$_2$. This is because there is a large intensity of the magnetic field at the interface between Ce:YIG and Si layer as can be seen in Fig. 2.6.

Figure 2.7 shows calculation results of the nonreciprocal phase shift for different waveguide widths with 2-D structure. Here, we use the waveguide structure with Ce:YIG/Si/SiO$_2$ and the rib height is the same as the waveguide height, i.e., it is a channel waveguide. The simulation grid sizes are $\Delta x = 0.01 \, \mu m$ and $\Delta y = 0.01 \, \mu m$ to model the narrow waveguide width. As is the particular problem for the software, spin-off intensity peaks sometime appear on the magnetic fields at the waveguide corners as shown in Fig. 2.8. The semi-vector simulation derives the magnetic fields from the firstly calculated electric fields so
as to satisfy the Maxwell equation. They appear only when the simulation is carried with small grid size for a high-index-contrast waveguide. These must be eliminated when the nonreciprocal phase shift is calculated using the magnetic fields. As you can see in Fig. 2.7, the maximum nonreciprocal phase shift becomes small for narrower width. This means that the magnetic field distribution is broaden in the vertical direction due to the lateral optical confinement as the waveguide width is narrow. Consequently, we should calculate the nonreciprocal phase shift with 2-D structure instead of 1-D structure if the lateral optical confinement is strong in the waveguide structure.

![Graph showing calculated nonreciprocal phase shift for some waveguide structures.](image)

Fig.2.5 Calculated nonreciprocal phase shift for some waveguide structures. Dashed lines are calculated by solving eigenvalue equation with 1-D structure and Solid lines are calculated by perturbation theory with 2-D structure, at $\lambda = 1.55 \, \mu m$. 

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*Chapter 2: Nonreciprocal phase shift for magneto-optical isolator*
Fig. 2.6 Mode profiles of magnetic field calculated by semi-vector BPM.
(a) SiO$_2$/Ce:YIG/SGGG at $h = 0.48 \mu$m, (b) Ce:YIG/GaInAsP/InP at $h = 0.36 \mu$m, and (c) Ce:YIG/Si/SiO$_2$ at $h = 0.20 \mu$m. All the width and rib height are $w = 2.0 \mu$m and $r = 0.1 \mu$m, respectively.

Fig. 2.7 Calculated nonreciprocal phase shift for some waveguide width with the Ce:YIG/Si/SiO$_2$ structure, at $\lambda = 1.55 \mu$m.
Next, we compare the simulation methods to calculate the magnetic field distribution with 2-D structure. The mode solving via semi-vector BPM, full-vector BPM, and full-vector FEM are considered [9,11]. The full-vector simulations provide $H_x$ and $H_y$ fields which are distinguished major or minor component as their distributions. Typically, major and minor components are distributed at the center and the corner in the waveguide, respectively. The $H_x$ of major component is called quasi-TM mode or TM-like mode.

Figures 2.9 and 2.10 show the calculation results for comparison of the semi-vector BPM with the full-vector BPM and the full-vector FEM, respectively. The waveguide is composed of SiO$_2$/Ce:YIG/SGGG and is a channel waveguide $h=r$. The magnetic fields are calculated directly in the full-vector simulations where no spin-off peaks appear as shown in Fig. 2.11. The semi-vector BPM shows larger maximum nonreciprocal phase shift than the full-vector simulations for small dimension of the waveguide. At the same time, the semi-vector BPM gives larger propagation constant than the full-vector simulations. This difference is thought to be caused by the spin-off peaks at the waveguide corners as mentioned above. The peaks
seem to prevent the field distribution from broadening to upper region as can be seen in Fig. 2.11 (a). This results in large distribution of the magnetic field at the magneto-optic interface and enhances the nonreciprocal phase shift.

Fig. 2.9 Comparison between semi-vector BPM and full-vector BPM. The lines are calculated by the semi-vector BPM, and the dots are calculated by the full-vector BPM.

Fig. 2.10 Comparison between semi-vector BPM and full-vector FEM. The lines are calculated by the semi-vector BPM, and the dots are calculated by the full-vector FEM.
Fig. 2.11 Mode profiles of magnetic field with the SiO₂/Ce:YIG/SGGG structure at $w=0.40\,\mu m$ and $h=0.24\,\mu m$ calculated by (a) semi-vector BPM, (b) full-vector BPM, and (c) full-vector FEM.

The discontinuity of the data for $w=1.0\,\mu m$ in Fig. 2.10 is due to hybrid supermodes. To understand the hybrid supermodes, effective refractive indices and nonreciprocal phase shift are calculated at the fixed height of $0.28\,\mu m$ as shown in Fig. 2.12 and Fig. 2.13. The mode-0 corresponds to the TE₀ mode. On the other hand, as the width increase, the mode-1 changes from TM₀ mode into TE₁ mode, and the mode-2 changes from TE₁ mode into TM₀ mode across $w \sim 1.0\,\mu m$. The nonreciprocal phase shift is calculated with $H_x$ component considered as TM₀ mode. The magnetic field distributions around the cross point of mode-1 and mode-2 are shown in Table 2.2. At $w=1.0\,\mu m$, the $H_x$ and $H_y$ can not be distinguished as major or minor component, then they are considered as hybrid supermodes. This behavior of the mode transition affects the field distributions at $w \sim 1.0\,\mu m$. For example, $H_{x1}$ at $w=0.95\,\mu m$ and $H_{x2}$ at $w=1.0\,\mu m$ are slightly pushed up and down, respectively. Therefore, the nonreciprocal phase shift calculated from the field distributions becomes discontinuous across $w \sim 1.0\,\mu m$. 
As a result, we should not design a waveguide with such parameter as hybrid supermode appears because they induce mode transition between TM fundamental and TE higher-order modes. Such mode transition is observed during the full-vector BPM simulation when the propagation constant of TM$_0$ is close to that of TE$_1$.

![Graph showing effective refractive indices](image1)

**Fig. 2.12** Effective refractive indices simulated by semi-vector BPM and full-vector FEM with the Ce:YIG/Si/SiO$_2$ structure at $h = r = 0.28$ μm.

![Graph showing nonreciprocal phase shift](image2)

**Fig. 2.13** Calculated nonreciprocal phase shift using the magnetic fields simulated by semi-vector BPM and full-vector FEM.
Table 2.2 Mode profiles of magnetic field calculated by full-vector FEM with the Ce:YIG/Si/SiO₂ structure at $h = r = 0.28 \mu m$.

<table>
<thead>
<tr>
<th>$w$</th>
<th>0.9 $\mu m$</th>
<th>0.95 $\mu m$</th>
<th>1.0 $\mu m$</th>
<th>1.05 $\mu m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{x1}$</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
</tr>
<tr>
<td>$H_{y1}$</td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
</tr>
<tr>
<td>$H_{x2}$</td>
<td><img src="image9.png" alt="Image" /></td>
<td><img src="image10.png" alt="Image" /></td>
<td><img src="image11.png" alt="Image" /></td>
<td><img src="image12.png" alt="Image" /></td>
</tr>
<tr>
<td>$H_{y2}$</td>
<td><img src="image13.png" alt="Image" /></td>
<td><img src="image14.png" alt="Image" /></td>
<td><img src="image15.png" alt="Image" /></td>
<td><img src="image16.png" alt="Image" /></td>
</tr>
</tbody>
</table>

2.3 Principle of isolator operation

In this section, the principle of a magneto-optical isolator operation employing nonreciprocal phase shift is described. Figure 2.14 shows the basic configuration of the isolator which is a Mach-Zehnder interferometer with nonreciprocal and reciprocal phase shifters. Hence we utilize the nonreciprocal phase shift for TM mode, the isolator operates for TM mode. External magnetic fields are applied in anti-parallel direction to the two arms of the nonreciprocal phase shifter. These provide direction-dependent propagation constants for the lightwave traveling in each arm. The nonreciprocal phase differences $\theta_N$ between the two arms are given by

\[
\theta_{N(\text{forward})} = (\beta_{1} - \beta_{2}) L, \quad (2.48)
\]

\[
\theta_{N(\text{backward})} = (\beta_{b1} - \beta_{b2}) L, \quad (2.49)
\]
where \( L \) is the propagation length of the nonreciprocal phase shifter. When the waveguide structures of the two arms are symmetric, the \( \theta_N \) has different sign depending on the propagation direction

\[
\beta_0 - \Delta \beta / 2 = \beta_{f1} = \beta_{b2}, \quad (2.50)
\]
\[
\beta_0 + \Delta \beta / 2 = \beta_{f2} = \beta_{b1}. \quad (2.51)
\]

In the isolator operation, \( \theta_N \) are set at \(-\pi/2\) and \(+\pi/2\) in the forward and backward directions, respectively. The reciprocal phase difference \( \theta_R \) is set at \(+\pi/2\) by an optical path difference \( \Delta L \) between the two arms given by

\[
\theta_R = \beta \cdot \Delta L \quad (2.52)
\]

where \( \beta \) is the longitudinal propagation constant of the mode propagating in the waveguide of reciprocal phase shifter.

A forward traveling wave from the input port is divided into two waves with identical amplitudes and phases at the 3dB coupler. The nonreciprocal phase difference is cancelled by the reciprocal one and the total phase difference is 0. So the two waves are coupled to the output port by the constructive interference at the 3dB coupler. A backward traveling wave from the output port is divided into identical two waves at the 3dB coupler. In this time, the nonreciprocal phase shift is added to the reciprocal one and the total phase difference is \(+\pi\). Then the two waves are not coupled to the input port and radiated at the 3dB coupler by the destructive interference. Consequently, the MZI functions as a magneto-optical isolator. Table 2.3 shows designed length of the nonreciprocal and reciprocal phase shifters for some waveguide structures. The refractive indices as shown in the Table 2.1 are used.

The output intensity of a MZI is given by [12]

\[
|E_{\text{out}}|^2 = \left| \frac{-j}{2} \left( \sqrt{a_1} e^{i\theta_1} + \sqrt{a_2} e^{i\theta_2} \right) E_{\text{in}} \right|^2
\]
\[
= \frac{1}{4} \left[ \left| a_1 + a_2 + 2 \sqrt{a_1 a_2} \cos(\theta_2 - \theta_1) \right| E_{\text{in}} \right|^2 \quad (2.53)
\]

where \( E_{\text{in}} \) and \( E_{\text{out}} \) are the amplitudes of the electric field at the input and output ports and \( \theta_1, \theta_2, a_1, \) and \( a_2 \) are the phase differences and attenuations of intensity in each arm. As the 3 dB coupler divided a wave into two waves with identical phase and amplitude, and the waves
propagate without any attenuation in both arms, the (2.53) is rewritten as

\[ |E_{\text{out}}|^2 = \frac{1 + \cos(\theta_2 - \theta_1)}{2} |E_m|^2. \quad (2.54) \]

\[ \theta_\text{N} \] \hspace{1cm} \[ \theta_\text{R} \]

Forward \hspace{1cm} -\pi/2 \hspace{1cm} +\pi/2 \\
\hspace{2cm} +\pi/2 \hspace{1cm} +\pi/2 \hspace{1cm} \text{Backward}

Fig. 2.14 Basic configuration of the magneto-optical isolator employing nonreciprocal phase shift. NPS: Nonreciprocal phase shifter, RPS: Reciprocal phase shifter.

Table 2.3 Designed lengths of nonreciprocal and reciprocal phase shifters.

\( \Theta_\text{F} = -4500 \text{ deg/cm of Ce:YIG at } \lambda = 1.55 \mu\text{m}. \)

<table>
<thead>
<tr>
<th>Structure</th>
<th>Parameters ( w / h / r ) (( \mu\text{m} ))</th>
<th>( L_\text{NPS} ) (mm)</th>
<th>( L_\text{RPS} ) (( \mu\text{m} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>SiO(_2)/Ce:YIG/SGGG</td>
<td>2.0 / 0.48 / 0.10</td>
<td>0.840</td>
<td>0.196</td>
</tr>
<tr>
<td>Ce:YIG/GaInAsP/InP</td>
<td>2.5 / 0.36 / 0.16</td>
<td>3.498</td>
<td>0.121</td>
</tr>
<tr>
<td>Ce:YIG/Si/SiO(_2)</td>
<td>2.0 / 0.20 / 0.01</td>
<td>0.213</td>
<td>0.164</td>
</tr>
<tr>
<td>Ce:YIG/Si/SiO(_2)</td>
<td>0.6 / 0.22 / 0.22</td>
<td>0.233</td>
<td>0.168</td>
</tr>
</tbody>
</table>

(a) semi-vector BPM, (b) full-vector FEM
2.4 Summary

In this chapter, the nonreciprocal phase shift is analyzed by numerical calculation. Using 1-D and 2-D waveguide models, the nonreciprocal phase shift is calculated. For narrower waveguide, 2-D model should be used for taking the effect of the lateral optical confinement into account. In the comparison among the several mode solving methods, reliable results are obtained by using full-vector simulations than semi-vector BPM. However, the full-vector simulations take much time to obtain the filed distributions and make it complex due to the hybrid supermode.
Chapter 2: Nonreciprocal phase shift for magneto-optical isolator

References


