Appendix A

Geometrical Interpretation of Update Vectors Perturbed by Outlier

We consider the perturbation that the update vectors of EMAPA, presented in Chapter 5, the second order APA, and NLMS suffer from an outlier. Here, we suppose the case where the microphone signal $y(n)$ includes an outlier $a(n)$, as supposed in 5.3.3. The update vector (5.14) given by EMAPA can be rewritten as

$$\Delta \hat{h}_{\text{EMAPA}}(n) = \Delta \tilde{h}_{\text{EMAPA}}(n) + \frac{a_1(n)}{r^T(n)x(n)} \cdot \frac{r(n)}{\|r(n)\|}.$$  \hspace{1cm} (A.1)

The update vector (5.7) given by APA can be rewritten as

$$\Delta \hat{h}_{\text{APA}}(n) = \Delta \tilde{h}_{\text{APA}}(n) + \frac{a_1(n)}{\|u(n)\|} \cdot \frac{u(n)}{\|u(n)\|} + \frac{a_2(n)}{\|w(n)\|} \cdot \frac{w(n)}{\|w(n)\|}.$$  \hspace{1cm} (A.2)

And the update vector given by NLMS [1] can be rewritten as

$$\Delta \hat{h}_{\text{NLMS}}(n) = \Delta \tilde{h}_{\text{NLMS}}(n) + \frac{a_1(n)}{\|x(n)\|} \cdot \frac{x(n)}{\|x(n)\|}.$$  \hspace{1cm} (A.3)

Here, we apply $\delta = 0$ to (5.7) and (5.14) for the simplicity. The vectors $r(n)$ and $w(n)$ used in the above are

$$r(n) = x(n) - \mu r_0(n-1) x(n-1) \hspace{1cm} (A.4)$$

$$w(n) = x(n-1) - \frac{r_1(n)}{r_0(n)} x(n). \hspace{1cm} (A.5)$$
Appendix A. Geometrical Interpretation of Update Vectors

Perturbed by Outlier

The vector \( \mathbf{u}(n) \) is given in (5.19), and the outliers are rewritten as \( a_1(n) = a(n) \) and \( a_2(n) = a(n - 1) \). And \( \Delta \hat{h}_{\text{EMAPA}}(n) \), \( \Delta \hat{h}_{\text{APA}}(n) \), and \( \Delta \hat{h}_{\text{NLMS}}(n) \) are the true update vectors of EMAPA, APA, and NLMS, respectively, which are obtained without the outliers:

\[
\Delta \hat{h}_{\text{EMAPA}}(n) = \frac{\tilde{e}_1(n) \| \mathbf{r}(n) \|}{\| \mathbf{r}(n) \|} \frac{\mathbf{r}(n)}{\mathbf{x}(n)} \cdot \frac{\mathbf{x}(n)}{\| \mathbf{x}(n) \|},
\]

\[
\Delta \hat{h}_{\text{APA}}(n) = \frac{\tilde{e}_1(n) \| \mathbf{u}(n) \|}{\| \mathbf{u}(n) \|} \frac{\mathbf{u}(n)}{\| \mathbf{u}(n) \|} + \frac{\tilde{e}_2(n) \| \mathbf{w}(n) \|}{\| \mathbf{w}(n) \|} \frac{\mathbf{w}(n)}{\| \mathbf{w}(n) \|},
\]

\[
\Delta \hat{h}_{\text{NLMS}}(n) = \frac{\tilde{e}_1(n) \| \mathbf{x}(n) \|}{\| \mathbf{x}(n) \|} \frac{\mathbf{x}(n)}{\| \mathbf{x}(n) \|},
\]

where

\[
\tilde{e}_1(n) = e(n) - a(n) \quad \text{and} \quad \tilde{e}_2(n) = (1 - \mu) e(n - 1) - a(n - 1).
\]

Figure A.1 shows the deviation ranges of filter update vectors \( \Delta \hat{h}_{\text{EMAPA}}(n) \), \( \Delta \hat{h}_{\text{APA}}(n) \), and \( \Delta \hat{h}_{\text{NLMS}}(n) \), according to the influence of outliers described in (A.1), (A.2), and (A.3). Here, we introduce the positive parameters \( \sigma_1 \) and \( \sigma_2 \) to specify the bound of the ranges:

\[
-\sigma_1 \leq \frac{a_1(n)}{\| \mathbf{x}(n) \|} \leq \sigma_1 \quad \text{and}
\]

\[
-\sigma_2 \leq \frac{a_2(n)}{\| \mathbf{x}(n - 1) \|} \leq \sigma_2.
\]

The deviations of the vectors \( \Delta \hat{h}_{\text{EMAPA}}(n) \) and \( \Delta \hat{h}_{\text{NLMS}}(n) \) are on the given straight lines parallel to the vectors \( \mathbf{r}(n) \) and \( \mathbf{x}(n) \), respectively, both of which are segmented depending on \( \sigma_1 \) only. On the other hand, The deviation of the vector \( \Delta \hat{h}_{\text{APA}}(n) \) is in the parallelogram whose sides are parallel to the vectors \( \mathbf{u}(n) \) and \( \mathbf{w}(n) \) and segmented depending on both \( \sigma_1 \) and \( \sigma_2 \). Therefore, the update vectors of EMAPA and NLMS are only perturbed in the magnitude by the outliers. And the update vector of APA are perturbed in both the magnitude and direction by the outliers. Note that Fig. A.1 is represented supposing the case where both \( \tilde{e}_1(n) \) and \( \tilde{e}_2(n) \) are positive.

By the way, \( \sigma_1 \) and \( \sigma_2 \) in respectively (A.11) and (A.12), which specify the bound of the deviation ranges, depend on the outlier-to-reference ratio (ORR). In case of the non-stationary reference input such as speech, the ORR varies dynamically. It can be often happen that \( \| \mathbf{x}(n) \| \) becomes larger than \( \| \mathbf{x}(n - 1) \| \) at a certain time index \( n \). So \( \sigma_2 \) can be significantly larger
Figure A.1: Deviation ranges of filter update vectors.
than $\sigma_1$, even if the outlier is stationary. In such a case of time index $n$, while EMAPA and NLMS are not perturbed by the outlier because of small $\sigma_1$, APA still suffers significant perturbation from the outlier because of large $\sigma_2$. Therefore, the robustness of APA is inferior to those of EMAPA and NLMS, as the ORR varies significantly due to the non-stationarity of the reference or outliers.
Appendix B

List of Publications, Presentations, and Patents

Journal Articles Related to This Dissertation


Other Journal Article

1. A. Nakagawa, S. Shimauchi, Y. Haneda, S. Aoki, and S. Makino, “A design of a hands-free communication unit using loudspeakers and mi-

Reviews


Letter


International Conferences


Technical Reports


Domestic Conferences


Registered Foreign Patents


Registered Japanese Patents


Appendix B. List of Publications, Presentations, and Patents


Bibliography


